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13. ABSTRACT *(Maximum 200 words)*

Substantive results were attained in applying distributions to reliability estimates and maintenance measures. A number of articles in national journals on topics such as "Simultaneous Confidence in Levels and Goodness of Fit for Failure Rates."

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Final Scientific Report to Air Force
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Grant AFOSR-89-0083

On the Reliability of Systems Subject to Maintenance and Repair

Department of Mathematics
The City College, CUNY
New York, NY 10031

Mark Brown, Principal Investigator

April, 1992

Grant AFOSR-89-0083 was awarded for the period 11/15/88 - 11/14/89

It was subsequently extended untio 1/14/92. This report will discuss the research accomplishments during the grant period.

1. A technical report "Weighted Sup-Norm Inequalities and their Applications," City College, CUNY Report No. MB89-02, AFOSR Technical Report 89-02, was issued in February 1989.

In this report a new method is developed for studying $sup_t(\alpha(t)|\bar{F}_1(t) - \bar{F}_2(t)|)$ where in applications, α is an increasing function, F_1 is a difficult to compute distribution of interest, and F_2 a simple computable approximation. By having $\alpha(t)$ increase and approach infinity as t increases, we get more useful bounds for large t (when $\bar{F}_1(t)$ and $\bar{F}_2(t)$ are small) than we do with an unweighted sup norm approach. Applications are presented to time reversible Markov chains, and to the distribution of the waiting time for a pattern. A paper based on this report appeared in *Communications in Statistics*, 19 (1990), 4061-4082.

2. A technical report "On two problems involving partial sums", City College, CUNY Report No. MB89-01, AFOSR Technical Report 89-01, was issued in January 1989. In this report two expected waiting time results are derived for partial sums of i.i.d. positive integer-valued random variables.

   (i) The expected waiting time for the partial sum to be a multiple of k for the first time is k/b where b is the greatest common diversor of k and d, where d is the period of the distribution.

   (ii) For two partial sums of i.i.d. positive integer valued random variables, the smallest common partial sum is $D_{\mu_1,\mu_2}/d_1d_2$, where μ_1 and μ_2 are the underlying means, d_1 and d_2 the periods, and D is the least common multiple of d_1 and d_2.

A paper based on this report appeared in *Probability in the Engineering and Informational Sciences*, 3, 1989, 511-516.

3. The technical report "Consequences of Monotonicity for Markov Transition Functions", City College, CUNY Report No. MB89-03, AFOSR Technical Report 89-03, was issued in February 1990.

For an ergodic Markov chain in discrete or continuous time, temporal monotonicity of transition probabilities are shown to have connections to approximate exponentiality

of first passage time distributions, as well as to the study of strong stationary times and separation distance.

These results lead to inequalities on the total variation distance between the distribution of a Markov chain at time t and the steady state distribution. Inequalities are also obtained for the sup norm distance between the distribution function of certain first passage times and their approximating exponential distribution functions. These bounds depend only on the ration of the relaxation time to the mean first passage time.

This report has been accepted for publication, subject to revisions, to *Annals of Applied Probability.*

4. The technical report "The Distribution of Total Variation Distance, with Applications to Simultaneous Confidence Intervals", City College, CUNY Report No. MB89-04, AFOSR Technical Report 89-04, was issued in July 1990.

For a one parameter parametric family, $\{P_\theta, \theta \varepsilon \Omega\}$ the asymptotic total variation distance between the unknown distribution P_{θ_0}, and the estimated distribution $P_{\hat{\theta}}$ where $\hat{\theta}$ is the maximum likelihood estimator of θ_0 is obtained. This immediately leads to simultaneous confidence intervals for $\{P_{\theta_0}(A), A \varepsilon \beta\}$ where β is the collection of Borel subsets of the real line. Various examples are worked out in detail. A paper based ont he report will appear as an invited paper in *Computer and Operations Research.*

5. The techncical report "Spectral Analysis, without Eigenvectors for Markov Chains", City College, CUNY Report No. MB89-05, AFOSR Technical Report 89-05, was issued in September 1990.

The report presents the Lagrange-Sylvester polynomial interpolation approach to spectral analysis of finite diagonalizable matrices, and discusses its implications for Markov chains. In the case of skip free to the right chains, explicit expressions for the first passage time distributions and transition probabilities are derived. These depend on the eigenvalues of the probability transition or infinitesimal matrix, but are derived without computation of eigenvectors. A paper based on this report appeared in *Probability in the Engineering and Informational Science*, 5, 1991, 131-144.

6. The technical report "Chi-Square Goodness of Fit: A Failure Rate Perspective", joint with M.H. Flicker, City College Technical Report No. MB89-06, AFOSR Technical Report 89-06, was issued in December 1990.

Employing a failure rate approach, a test statistic $\tilde{X}^2$ is proposed for the classic goodness of fit problem. It is then shown that a variation of $\tilde{X}^2$ obtained by replacing observed variances by expected variances leads to the classical test statistics, $\tilde{X}^2$. This leads to a new interpretation of the chi-square test as well as a new proof of its asymptotic distribution. An examination of the possible advantages and disadvantages of $\tilde{X}^2$ relative to X^2 is also presented.

A paper based on this report appeared in *Proabability in the Engineering and Informational Sciences*, 5, 1991, 273-284.

7. The technical report "Inequalities for Rare Events in Time-Reversible Markov Chains I", joint with David J. Aldous, City College, CUNY Technical Report No. MB89-07, AFOSR Technical Report 89-07, was issued in June 1991.

   The new inequalities include:

   (i) A bound on the sup norm distance between the first passage time to a set A starting in steady state, and an exponential distribution with the same mean. The bound depends only on the ration of the relaxation time to the expected first passage time.

   (ii) A two sided bound for the proability of hitting A prior to time t, starting in steady state. This bound depends only on the ratio of the relaxation time to the mean quasi-stationary hitting time.

   (ii) An inequality for the stationary measure of those states whose expected waiting time to hit A exceeds the expected stationary waiting time by a specified amount.

   (iv) Bounds on the density function for the first hitting time to A starting in steady state.

This paper was presented by the Principal Investigator as an invited paper at the AMS-IMS Conference on Proabability Inequalities, held in Seattle in July 1991. It will appear in the IMS Monograph/Lecture Note Series.

8. The technical report "Inequalities for Rare Events in Time-Reversible Markov Chains II", joint with David J. Aldous, City College, CUNY Technical Report No. MB89-08, AFOSR Report 89-08, was issued in August 1991.

   The Poisson approximation for numbers of rare events which actually occur, and

the exponential approximation for the waiting time until first occurrence of a rare event, are useful throughout many areas of probability. This paper is a theoretical study of explicit bounds in these approximations, in the special setting of hitting times of stationary reversible Markov chains.

9. During the grant period I presented the following invited talks based on my AFOSR research.

   (i) FSU Anniversary Statistics Conference-March 1990, Tallahassee, FL.

   (ii) Joint National ORSA-TIMS meeting, Philadelphia, October 1990.

   (iii) Columbia University, joint Statistics, Business School seminar, November 1990.

   (iv) ORSA-TIMS Probability Conference, Monterey, CA, January 1991.

   (v) AMS-IMS Conference on Inequalities, Seattle, July 1991.

   (vi) Johns Hopkins University, Department of Mathematical Sciences Seminar, February 1991.

   (vii) University of Connecticut, Department of Statistics Seminar, October 1991.

   (viii) Columbia University, Department of Statistics Seminar. October 1991.

   (ix) CUNY Graduate Center, Department of Mathematics Seminar, April 1992.